

## NUMERICAL MODELING OF THE MECHANICAL BEHAVIOUR OF COMPOSITE MATERIALS

Schaeffer B

**ABSTRACT:** The mechanical behaviour of composite materials has been simulated numerically without analytical homogenization with Deform 2D, a fully non-linear bidimensional software working on a Macintosh microcomputer, taking advantage of the graphic user interface. Automatic meshing is performed with a simple click in a contour. The software is especially adapted to the study of impacts but quasi-static tests may also be simulated. Composite materials are modelled by distributing two component materials over the meshes according to a periodic structure described by a graphic pattern. Numerical and experimental results have been compared for tubes manufactured by filament winding. A Poisson's ratio larger than 0.5 was found numerically, in agreement with experiment. The "knee" on the stress-strain curve was also simulated.

**KEY WORDS:** finite differences, dynamics, composite material, software, fracture, numerical simulation, homogenization, elastic constants

### I. INTRODUCTION

Composite materials are characterized by an inhomogeneous structure. Even with isotropic component materials, at a macroscopic scale, the mechanical properties may be orthotropic if the distribution of the reinforcing isotropic component (for example glass fibers) is orthotropic. Complicated constitutive equations and fracture criterions are used in numerous homogenized models. The mechanical properties of the composite have to be either calculated or experimentally established before any structural analysis. This step should be integrated in numerical computations like finite element analysis. This approach has been mainly used for isolated fibers or locally in a unit cell of a periodic structure [1, 2].

Why not using it for the whole composite structure? The easiest way to do it is to consider that the composite is a structure made of two or more isotropic materials having elementary constitutive equations and fracture criterions. Of course it is not possible to have as many elements as there are fibers, they will be much larger than the real fibers. The process of homogenization will be automatically performed in the numerical calculation without having to calculate separately the mechanical properties of the composite.

Homogenization theories have their origin in the application of anisotropic elasticity of crystalline materials to composites. For example, Hill's theory on plasticity of slightly orthotropic alloys has no relevance to the strength of conventional fiber/polymer composites [3].

Analytical homogenization is not used in this paper: the isotropic component materials are distributed throughout the meshes, each made of a single component material. The gross behaviour of the material is obtained as the result of a complex structure made of two different materials.

## II. NUMERICAL METHOD

Deform2D is a finite differences software where the specimen to be studied is divided into elementary cells, e.g. quadrilaterals (figure 1) [4,5]. The material may be different from one cell to the other. This model is very similar to usual homogenized models [1,6] but the homogenization is done numerically instead of being done analytically. The structure is defined by a graphical pattern associating one cell to one pixel.

A cell is a rectangle (or a quadrilateral along the boundary) made of four triangular meshes. The cells become quadrilaterals after deformation. The material is the same over one cell. Stresses and strains are uniform in each triangle. The numerical experiment, for example a tensile test on a sample hold in a fixed grip at the bottom and in a moving grip at the top is shown on figure 1.

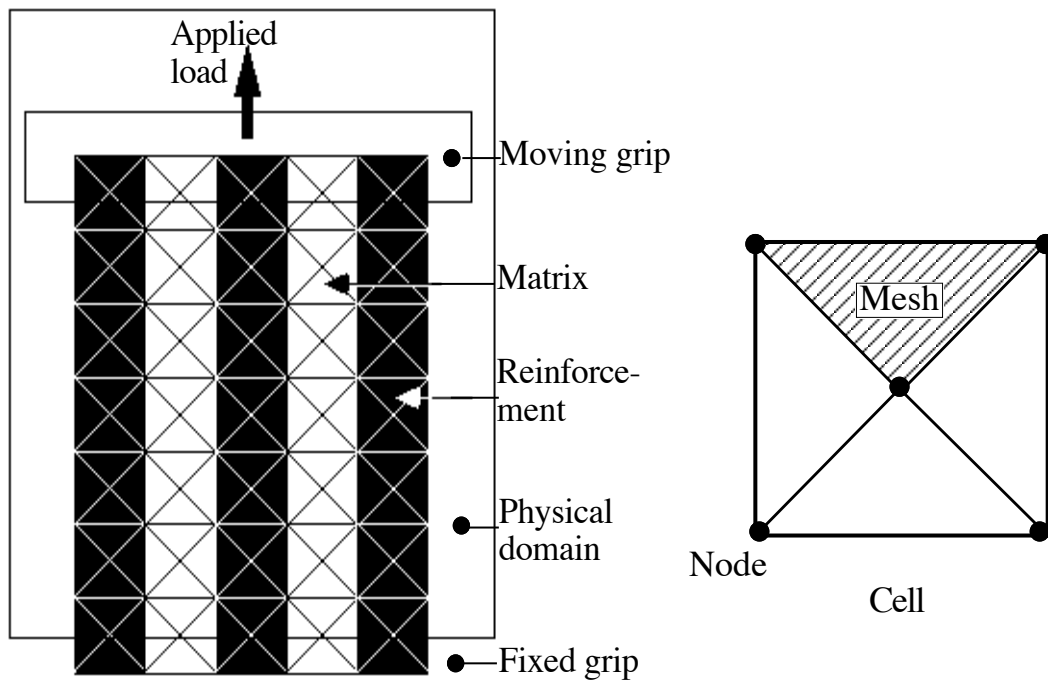


Fig.1 Mesh model of a tensile specimen of a fiber reinforced material.

The finite strain tensor is computed from the lengths of the sides of a triangle. The stresses are obtained from the strains through the constitutive laws of the material of the cell, a combination of isotropic, hypoelastic, viscous and plastic (with upper and lower yield stresses) behaviour and a Coulomb-Tresca type fracture criterion. The strength of the interfaces is not taken into account as such: fracture is assumed to take place either in the matrix or the fiber. The resulting effort on a node is then calculated. Newton's law gives the acceleration, integrated twice, to obtain the new coordinates of the node.

The computing cycle is repeated for each triangle and each node at each time step. The Courant-Friedrich-Lewy criterion states that, for stable computation, the propagation of the elastic wave has to be smaller than one mesh during one time step. More details may be found elsewhere [4,5].

Engineering elastic constants for each component material and other experimental data are entered in a dialog window. Stresses, strains, displacements etc... may be visualised and the calculated picture may be transferred in any Macintosh software. The programming language is C++.

### III. INFLUENCE OF FIBER ORIENTATION ON FRACTURE

#### 1. Experimental study

Tubes were manufactured by filament winding of fiberglass with epoxy resin. The range of winding angles extents from  $15^\circ$  to  $80^\circ$ . The weight content of the fibers is between 55 and 65 % or between 40 and 50 % in volume. The mechanical properties were measured by radial pressurization of tubes, that is in uniaxial tension. Diametral compression tests and pressurization tests with axial and radial stresses were also performed but are not reported here. The elastic constants, measured with strain gages, had been compared [7] with those obtained with Puck's homogenization theory. The purpose of this paper is to integrate the process of homogenization in the numerical simulation.

#### 2. Numerical experiments

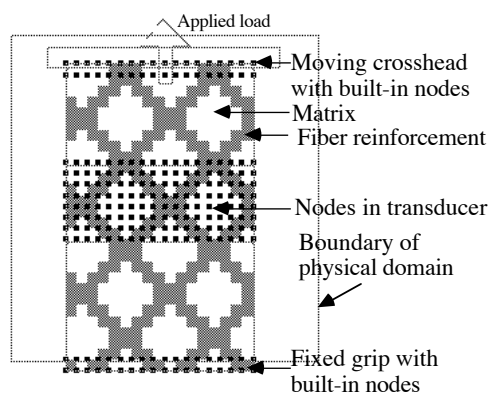


Fig.2 Model of tension specimen for 45° winding (44 % volume content of fibers).

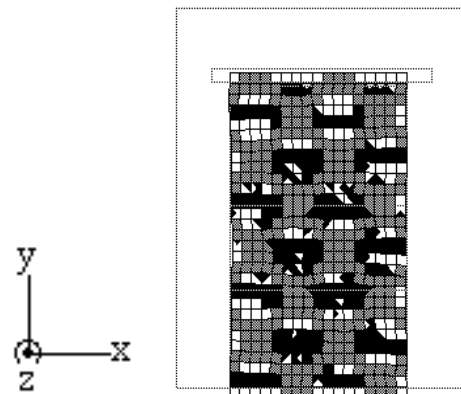


Fig.3 Reinforcement (in gray) and cracks (in black) for a 50 % volume content of fibers wound at 45° at maximum stress.

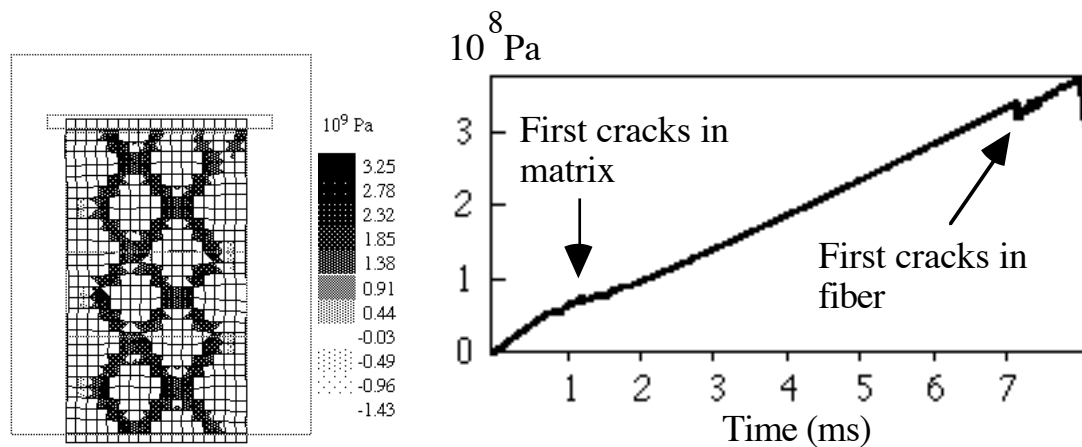


Fig.4 Stress distribution at fracture in a 45° wound specimen (50 % volume content of fibers).

Fig.5 Evolution of the mean vertical stress  $\sigma_{yy}$  in the centre of the specimen in tension (50 % volume content of fibers). The curve shows a "knee" often observed experimentally [6].













The numerical experiment is a tensile test on a plate of 0.1 m width by 0.2 m height. Plane stress is assumed. The speed of the moving crosshead is 1 m/s. This speed, much larger than usual speeds in order to minimize the duration of the calculation, is nevertheless quasi-static. The number of nodes is 500. The elastic constants and the tensile strengths of glass and epoxy resin are given in the table below. The specific mass is respectively 1,200 and 2,600 kg/m<sup>3</sup> for resin and glass.

The load extension (time) curve is linear until fracture for glass, resin, horizontal and vertical fibers. For 45° oriented fibers, a non-linear behaviour is obtained, due to cracking in the matrix, fracture is progressive until fiber cracking occurs (fig.5). The shape of the curve is very similar to those observed experimentally by Puck [6].

### 3. Comparison with experiment

The following table shows the comparison between simulation and experiment for Young's modulus, Poisson's ratio and ultimate tensile strength. The composite structure is modelled by various structures represented by graphic patterns approximating winding modes. For comparison the results of simulated tensile tests on pure glass and resin are also given.

**Table :** Young' modulus E , Poisson's ratio  $\nu$  and ultimate tensile strength, simulation and experiment.

Structure			E GPa		$\nu$		Maximum stress MPa	
Winding angle	Volume % glass	Structure (pattern)	Simulated	Experimental	Simulated	Experimental	Simulated	Experimental
Glass	100		71	75	0.25	0.25	3300	3400
90	50		35		0.21		1600	
73	50		34	35±5	0.28	0.2±0.1	1200	900±100
73	50		30		0.42		860	
73	50		22		0.36		700	
45	50		15		0.51		450	
45	44		13	11±3	0.65	0.6±0.2	420	300±100
45	50		13		0.44		370	
27	50		18		0.13		600	
27	50		16	7±3	0.30	0.1±0.05	310	50±10
27	50		11		0.21		310	
14	50		12		0.09		100	
0	50		5		0.05		47	
Resin	0		3	2.7	0.39	0.37	42	50

For the 45° winding, with 44 % volume content of glass, one may notice that Poisson's ratio from the numerical simulation is 0.65, larger than 0.5, not far from the experimental value of 0.6. This high value of Poisson's ratio may be explained by the parallelogrammatic deformation due to the crossed inclined fibers. For the two 50 % volume content, the intersection of the fibers is reinforced and, therefore the parallelogrammatic deformation is inhibited and Poisson's ratio is lower than for the 44 % volume content.

The strength increases regularly with increasing winding angle except when the pattern corresponds to a more rigid structure. For vertical fibers, the calculated strength is half the glass strength as may be expected from the "rule of mixtures".

Young's modulus and ultimate tensile strengths are larger by simulation. It may be due to a larger volume content of reinforcement (generally 50 % in volume) in the

simulation than in the real material. Perhaps, due to poor bonding between fiber and resin, one should use a lower resin strength for the composite than for pure resin.

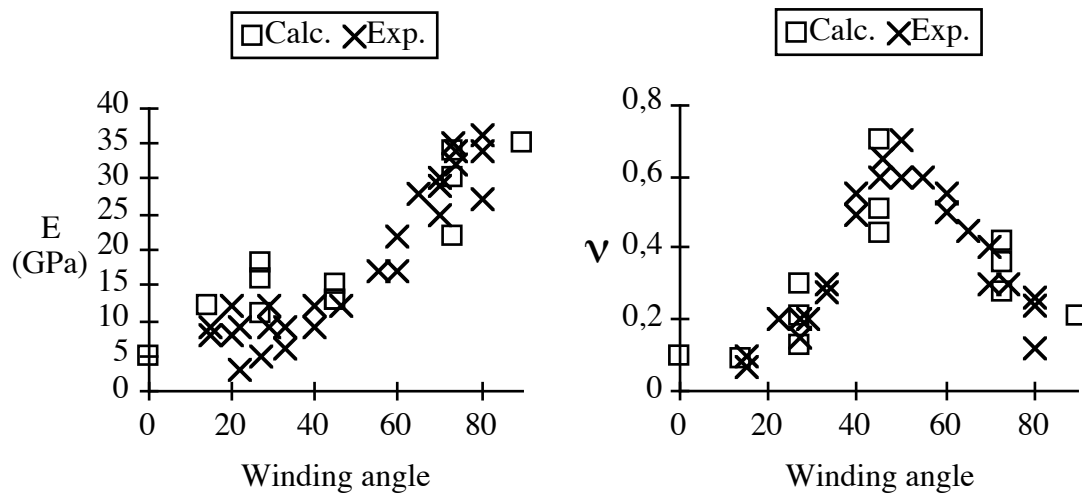


Fig.6 Elastic constants  $E$  and  $\nu$ , simulated and experimental.

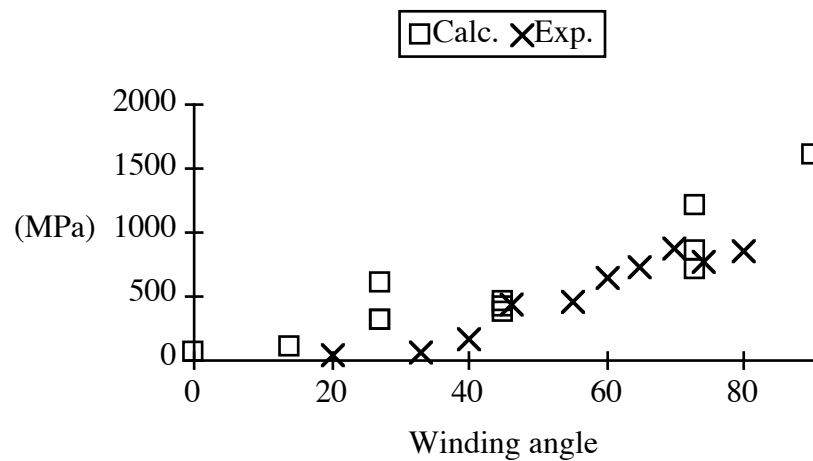


Fig.7 Ultimate tensile strength, simulated and experimental.

Although calculated moduli and strengths are higher than found experimentally, the difference between calculated and experimental data is almost within the experimental scatter.

#### IV. CONCLUSIONS

This paper shows that numerical homogenization may replace analytical homogenization by using directly in the software the physical properties of the component materials without having to calculate separately the elastic constants of the composite. It is only necessary to give the structure of the composite material and the physical constants of each component. A variety of micromechanical structures may be used but is limited to the patterns available in a 8 by 8 bit image.

The agreement between simulation and experiment is near statistical signification but not worse than with other homogenized models where it is always necessary to adjust the numerical constants to the properties of the real composite. The well-known “knees” on the stress-strain curve of fibrous composites have been simulated.

It would be interesting to apply this simulation to the compressive behaviour of fibrous composites but we have not done true compression tests except diametral compression of tubes where fracture occurs in bending.

The idea proposed in this paper was to use numerical method “ab initio” from the properties of the fibers and the matrix without using intermediate layers or other homogenized microstructures, difficult to measure experimentally and subject to large variations. It has been applied in two dimensions but should be possible also in three dimensions.

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