

## **Numerical Modelling of Fast Crack Propagation on a Microcomputer**

B. Schaeffer

### **ABSTRACT**

Many experimental and theoretical fast crack propagation studies have been performed during the last decades. This paper describes numerical experiments using Deform 2D, a bidimensional software working in non-linear dynamics on a microcomputer<sup>1</sup>. Rheological behaviour may be elastic, plastic, viscous, compressible, weighty. Plasticity may be with or without upper yield stress, with or without strain-hardening. All these properties may be combined. Propagation of waves, glide bands and cracks have been visualised. A graphic user interface and an interface with standard Macintosh design software has been developed. Meshing is done automatically by clicking in the contour of the specimen. A cut is necessary for multiply connected geometries.

The numerical simulation of fast crack propagation is compared with experiments<sup>2</sup> on brittle plastics such as Homalite. The fracture criterion used is the Coulomb criterion. The tensile strength was adjusted to obtain the desired critical stress-intensity factor. The crack being defined by the cracked zone, with a thickness at least equal to the mesh size, a large number of meshes is necessary. Computations were performed on a single-edged-notched tensile specimens excentrically pin-loaded. The crack speed was found to have an approximately constant value of two thirds of the shear wave velocity, twice the experimental value. The computations were not precise enough to determine any relationship between speed and stress-intensity factor. Some crack bifurcation occurred, particularly on the moving side of the crack, but complete bifurcation was not obtained. A more detailed analysis and testing of the equations is under way to find a slightly better formulation.

## INTRODUCTION

With the advent of microcomputers, numerical experimentation has been accessible to small companies for the price of an automobile. Though microcomputers have growing computing power, large computers remain necessary for complex structures. The proposed method is especially convenient for simulating laboratory tests.

The computing method is based on dividing the domain into quadrilateral cells where stresses and strains are constant. Strains are determined directly from the variation of the geometry of the mesh during deformation. Stresses are related to strains using a constitutive law. Various conditions such as plasticity or fracture criteria or boundary conditions are checked. The force applied to an element obtained by joining the four immediate neighbours of the considered node is calculated from the stresses. By numerical integration of Newton's law, the position of the nodes at each time step is computed and the computing cycle is repeated a large number of times. The principle of the computation is to replace analytical solutions by repeated elementary calculations directly on the simple formulae expressing the basic laws of mechanics. No simplifying assumptions such as incompressibility or symmetry considerations are used. On the other hand, remeshing, artificial viscosity or sophisticated numerical methods are not necessary. It seemed preferable to use a large number of nodes than a complicated algorithm.

The problem of dynamic fracture is still unsolved. Two main questions are opened: the theoretically found crack speed is much higher than the experimental value and the assumed relationship between the speed and the stress intensity factor seems to be multivalued.

## THEORY

### Deformations

In the formulation of continuum mechanics the configuration of a solid body is described by a continuous mathematical model whose geometrical points are identified with the place of the material particles of the body<sup>3</sup>. An infinitesimal vector of length  $ds$ , with coordinates  $dx_i$ , becomes, after deformation, a vector of length  $ds'$ , with coordinates  $dx'_i$ .

With first order approximation, the new coordinates are linearly related to the initial coordinates through the deformation gradient matrix<sup>4</sup> :

$$ds'^2 - ds^2 = 2 \varepsilon_{ij} dx_i dx_j \quad (1)$$

where

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial x'_k}{\partial x_i} \frac{\partial x'_k}{\partial x_j} - \delta_{ij} \right) \quad (2)$$

are Green-Lagrange strains (finite strains). The strain tensor may be computed if one knows the lengths of the sides of a triangle by solving a linear system of three equations with three unknowns. Volume strain is computed from the area of the triangle. Therefore volumetric and shear strains are separated.

Applying formula (1) to the vector lying in the direction of the first coordinate axis, we find<sup>5</sup>.

$$\varepsilon = \frac{1}{2} \left[ \left( \frac{ds'}{ds} \right)^2 - 1 \right] \approx \frac{(ds' - ds)}{ds} \quad (3)$$

For small strains, this agrees with the classical definition of Cauchy infinitesimal strains<sup>3</sup> used by experimentators when, during a tensile test, they measure the length of the sample as a function of its initial length. The volume strain is approximated by

$$\frac{dV}{V} = \varepsilon_{mm} \quad (4)$$

### Stresses

The constitutive law or rheological equation of state<sup>6</sup> relates stresses and strains. The stress tensor will be the true stress (relative to the instantaneous geometry) and not the nominal or engineering stress (relative to the initial geometry). Indeed, Newton's laws of mechanics are applicable to the current shape of a body.

The stress tensor components  $\sigma_{ij}$  may be separated into spherical and deviator parts:

$$\sigma_{ij} = \frac{\sigma_{mm}}{3} \delta_{ij} + \left( \sigma_{ij} - \frac{\sigma_{mm}}{3} \delta_{ij} \right) \quad (5)$$

The constitutive law has also a spherical and a deviator part. The spherical part relates the pressure to the volume, it is what is currently called equation of state. A more general equation of state, the rheological equation of state, in differential form has been chosen. It is a linear combination of a hypoelastic solid and a viscous fluid:

$$\frac{d\sigma_{ij}}{dt} = K \frac{d\varepsilon_{mm}}{dt} \delta_{ij} + 2\mu \frac{d}{dt} \left( \varepsilon_{ij} - \frac{\varepsilon_{mm}}{3} \delta_{ij} \right) + 2\eta \frac{d^2}{dt^2} \left( \varepsilon_{ij} - \frac{\varepsilon_{mm}}{3} \delta_{ij} \right) \quad (6)$$

$K$  is the bulk modulus,  $\mu$  the shear modulus and  $\eta$  the viscosity.

This constitutive law may be completed by, for example, a plasticity criterion (Tresca) : the shear stress is limited by the maximum shear stress (half the uniaxial yield stress). The fracture criterion is defined in a similar way, but with a linear dependence of strength with pressure (Coulomb criterion).

### Movement

The motion of a region  $\Omega$  bounded by a surface  $\Sigma$  of the continuum is obtained by integrating Newton's law of motion

$$\iiint_{\Omega} \rho \Gamma_i dV = \iiint_{\Omega} \rho g_i dV + \iint_{\Sigma} \sigma_{ij} n_j dS \quad (7)$$

where  $\rho$  is the specific mass,  $\Gamma_i$  and  $g_i$  are the components of the accelerations of the centre of gravity of  $\Omega$  and of gravity.  $dV$  and  $dS$  are elementary volume and surface elements. The  $n_j$  are the components of the normal to surface  $\Sigma$ .

The domain  $\Omega$  being small, the density is constant and the components of the acceleration may be written

$$\Gamma_i = g_i + \frac{1}{M} \iint_{\Sigma} \sigma_{ij} n_j dS \quad (8)$$

where

$$M = \iiint_{\Omega} \rho \, dV \quad (9)$$

is the mass of the volume element  $\Omega$ . Knowing the value of the right side of equation (8), the movement of the centre of gravity of  $\Omega$  may be computed by a double quadrature that gives first its speed

$$V_i = \int_0^t \Gamma_i \, dt \quad (10)$$

and then its position

$$X_i = \int_0^t V_i \, dt \quad (11)$$

The volume element  $\Omega$  being small, the rotation around the centre of gravity may be neglected.

## NUMERICAL SOLUTION

The principle of discretization is represented on fig.1. The meshes, rectangular and identical on the figure, are in practice squares inside the specimen and quadrilaterals near the contour. In every mesh, the stresses and strains are constant. On the boundary, the meshes have zero stresses. The boundary conditions may be of given position or speed or pressure.

The strains are a linear function of the squares of the length of the sides of the reference triangle, built with the diagonals of the quadrilateral cells, as may be shown from equation (2):

$$\varepsilon_{ij} = a_{ij} + \sum_{k=1}^3 b_{ijk} L_k^2 \quad (12)$$

The  $a_{ij}$  and  $b_{ijk}$  coefficients may be computed once only, at program initialization or at each computing cycle. The results in this paper have been calculated with the second method (incremental) but it seems from earlier computations that better results with respect to crack

bifurcation have been obtained with constant  $a_{ij}$  and  $b_{ijk}$  coefficients. This formula is used for the deviator strains. Volumetric strain is computed directly from the area of the cell, given by the cross-product of the diagonals of the mesh.

Stresses are calculated from the strains using the rheological equation of state (6) . They are limited by the plasticity and fracture criterions. No special “elements” are used, the precision being simply limited by the mesh size and no assumption is made for the stresses at the crack tip except Tresca or Coulomb criterions.

The movement of a node, considered as a material point, is computed by applying Newton's law to the solid defined by joining together the four nodes surrounding the considered node. It is (on fig.1) a lozange made from four half cells. The resultant force on an element is the sum of the four surface forces applied along its sides. At each time step, the acceleration, given by equation (10), is integrated numerically by finite differences of the first order, to obtain the components of the speed. Integration of equation (11) gives the components of the coordinates of the node.

This computing cycle is repeated for each node and each time step. The Courant-Friedrich-Lewy criterion<sup>7</sup> stating that, for stable computation, the propagation of the wave has to be smaller than a mesh during one time step. The computing time is hence inversely proportional to the size of the mesh. For one cycle it is proportional to the number of meshes. The total computing time is then proportional to the 1.5 power of the number of nodes.

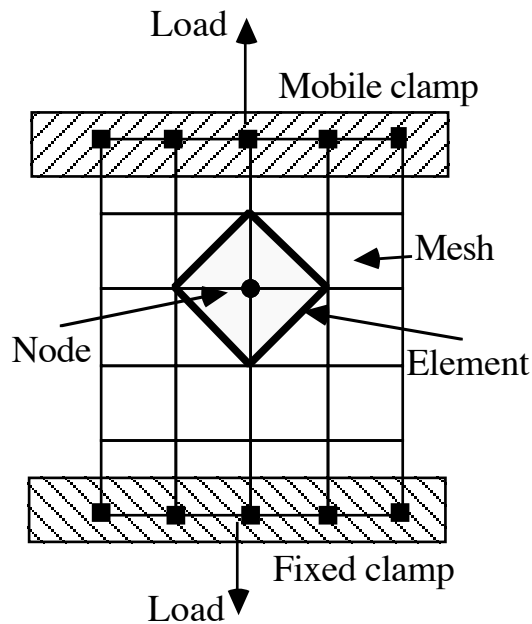


Figure 1. Geometry and grid for a schematic tension test.

## USER INTERFACE

The contours defining the problem geometry are created with the help of any Macintosh drawing program (fig.2). Three regions are to be defined by its contour:

- the deformable body, to be meshed.
- the physical domain where the nodes are allowed to move.
- the moving part, imposing a displacement to the deformable body.

Other contours may be introduced graphically: pressurised zone, initial speed zone, reinforced zone... Each mesh having individual physical properties, it is possible to study composite materials (two isotropic components).

Built-in conditions are obtained graphically if the region of the deformable body is not entirely contained in the physical domain or intersects the moving part. The same principle applies for other regions: the nodes or the meshes having given characteristics if they are in the given region. For example, nodes at the surface of the specimen that are in the pressurised zone have a given pressure. If meshes are entirely

contained in the pressurised zone the initial pressure is the given pressure. It is not necessary to define individual characteristics of the nodes or meshes, this is taken automatically by the program directly from the drawing. Changing the total number of nodes is done simply by changing the figure in a dialog window.

Contact is defined by testing if a node is or not inside the physical domain or the moving part. If there is contact, the speed of the node is made tangent to the contour.

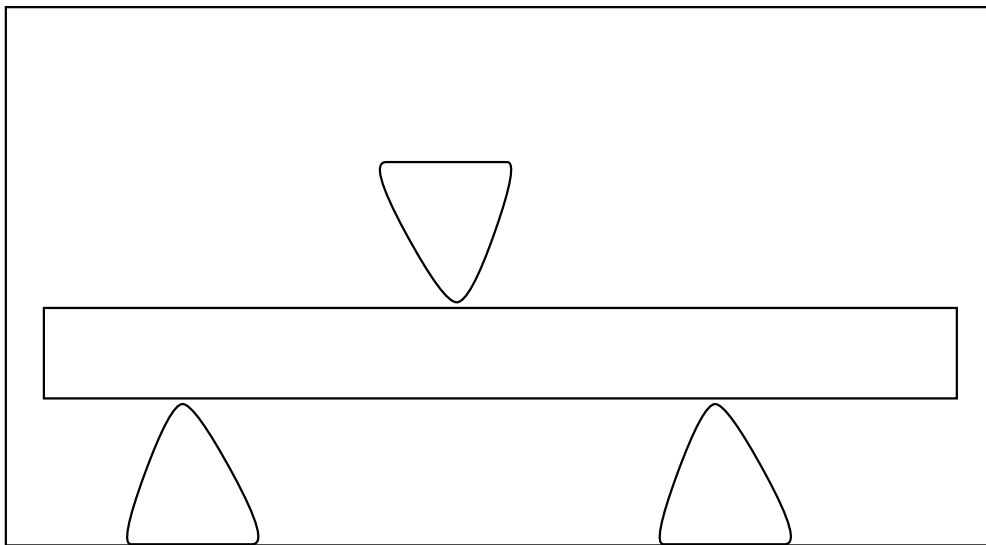


Figure 2. Graphical entry for a bending test.

The three zones are designated by successively clicking the mouse inside each contour, resulting in the creation by the program of a Macintosh “region” associated with each zone. For built-in conditions, it is necessary that the specimen’s region intersects either or both of the moving part and the exterior of the physical domain. The conditions of the numerical experiment (overall dimensions, ambient pressure, applied or initial pressure, applied or initial speed, acceleration, duration of the applied pressure or speed, total number of nodes) and the physical constants for two materials (bulk modulus, shear modulus, lower and upper yield stresses, tensile strength, strain at fracture for strain-hardening, internal Coulomb friction angle, viscosity, specific mass) are introduced through a printable dialog window. Other properties of the materials such as Young’s modulus, Poisson’s ratio, longitudinal and transverse wave speeds are computed from the preceding values and displayed in the window. The structure of the composite material is



visualised in the same window by means of a pattern chosen in an other dialog window.

Meshing is automatic, the meshes are squares except near the contour where the nodes are adjusted to it and become quadrilaterals. The software attributes to each mesh its own material and physical constants.

Pressure, stresses, plastic zones or cracks, applied load and total volume change may be visualised. The results of the computations may be recorded on disk in graphical form (vectorial or bitmap) and replayed as an animation. A camera may be triggered to record the screen and create a moving picture.

The software has been programmed in Pascal, first on an Apple II microcomputer, later on a Macintosh. A typical computing time is of about one day with 5000 nodes on a Macintosh II with 8 Mbytes of memory and System 7. One mesh necessitates about one kilobyte. In order to minimise computing times, it is necessary to use high rates of deformation.

## NUMERICAL EXPERIMENTS

### Geometry

The specimen (thickness 5 mm) has a single edge notch. It was not possible to obtain a sharp crack, the notch radius being limited by the mesh size (0.3 mm for 5000 nodes). The notch depth varied between 13 and 20 mm. The minimum size of the notch was limited by the fact that fracture occurred at the boundary of the reinforcement around the pin holes. instead of appearing at the notch root.

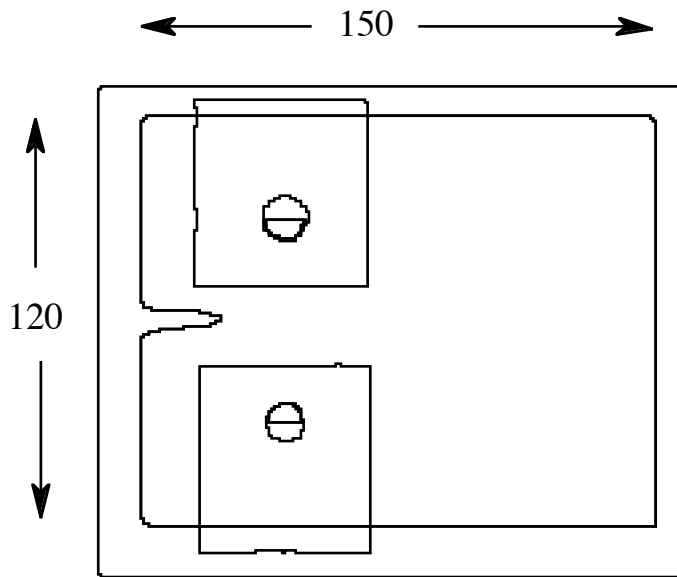


Figure 3 Graphical entry. The input of the specimen geometry is done by clicking in its contour. The overall dimensions are in mm. There are two holes for the moving and fixed pins. To obtain a simply connected region it is necessary to draw a line between each hole and the contour. The holes in the specimen are in fact half holes in order to have built-in conditions and avoid contact problems. The fixed and moving pins are full circles. The physical domain is described by the outer rectangle, excluding the fixed pin, from which a line is drawn to the outer rectangle to create a simply connected domain. The two rectangles around the pins show where the material is reinforced (high strength but otherwise identical material constants) to avoid cracking around the stress concentrations at the holes.

### Materials

The materials studied are Homalite 100, Homalite 911 (CR 39) and PMMA. using data from the literature<sup>2, 8</sup> In this paper, results on CR 39 only are given (Young's modulus: 4.1 GPa, shear modulus: 1.4 GPa, density 1300 kg/m<sup>3</sup>, tensile strength:  $R_t = 10$  MPa.). The critical stress-intensity factor  $K_{Ic}$  may be calculated to 0.3 MPa $\sqrt{m}$  from the tensile strength and a mesh size  $a_c$  of 0.3 mm (with 5000 nodes) using the formula :

$$K_{Ic} = R_t \sqrt{2 a_c} \quad (13)$$

It is near to the experimental value of 0.4. The maximum load (fig. 4) is found to be of 1200 N to be compared with the experimental value<sup>2</sup> of 1667 N. The crosshead speed is 1 m/s, much larger than in the real experiments (1 inch / min).

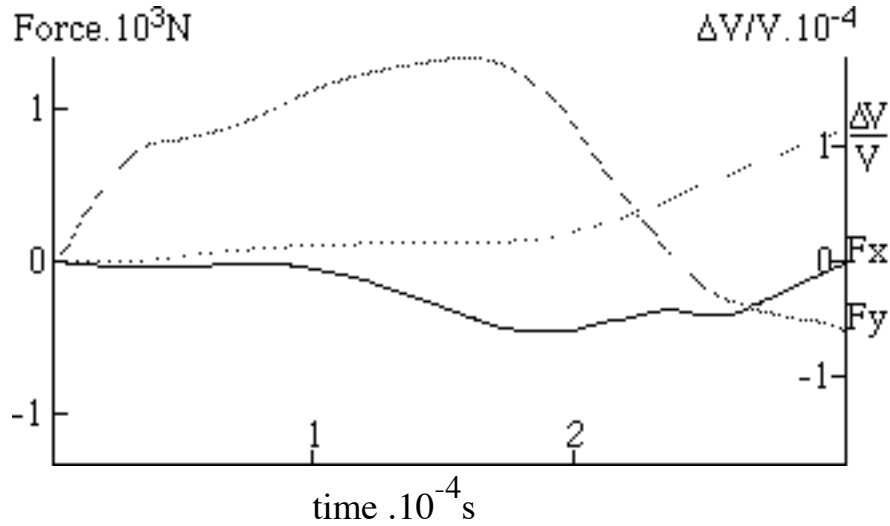


Figure 4 Vertical ( $F_y$ ) and horizontal ( $F_x$ ) components of the force applied to the specimen. The horizontal component of the force is not zero, this dissymetry may be due to reflected waves on the sides of the specimen. The volume change  $\Delta V/V$  increases rapidly at fracture due to the crack opening.

The stress distributions around the moving crack are represented on fig. 5. Fig. 6 shows the crack propagation sequence. Fig. 7 represents the mean stress intensity factor and the crack length, during crack propagation, calculated from measurements on the pictures of fig. 6. At each time step the stress intensity factor is computed from

$$K_I = (\sigma_{xx} + \sigma_{yy}) \sqrt{\frac{\pi r}{2}} \quad (14)$$

where  $r$  is the distance to the crack tip from the point of measurement. The crack speed may be almost constant at a value of 700 m/s, about twice the experimental value. There seems to be a maximum of speed around 220  $\mu$ s, coinciding with a maximum of  $K_I$ , as is observed experimentally<sup>2</sup> but it is to be confirmed with more precise calculations. Numerical experiments were done with a higher viscosity and also with a yield stress low enough to produce a plastic deformation, but no

decrease of the crack velocity was found. The crack velocity remains the same for a lower number of nodes (1000) that should give a higher fracture energy.

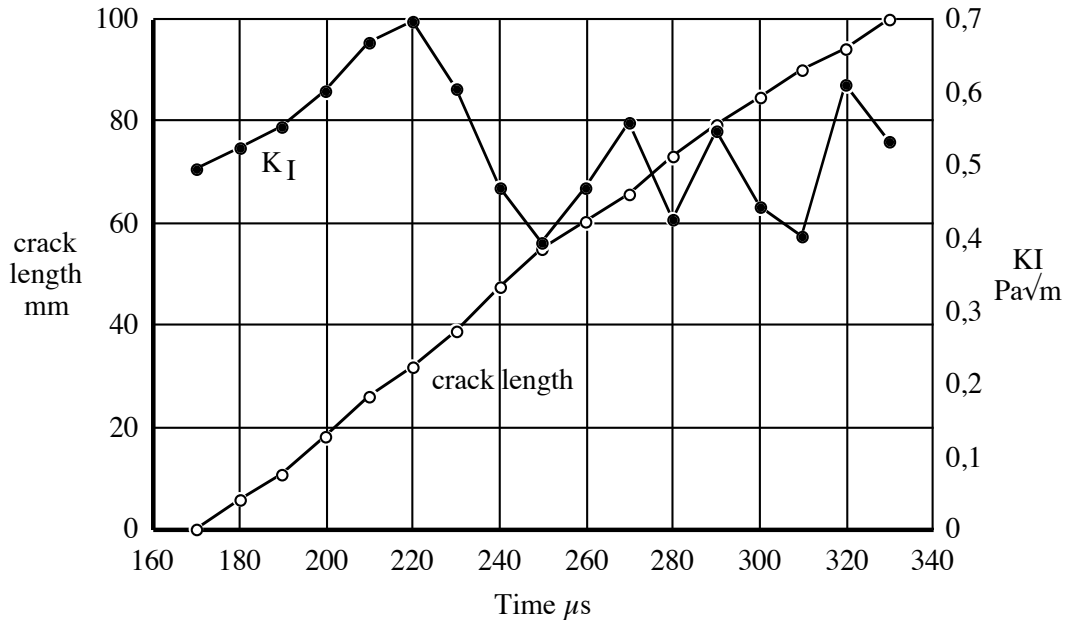


Figure 7 Stress-intensity and crack length from the data of fig. 6

## CONCLUSIONS

An explicit method for the simulation of the dynamic non-linear behaviour of two-dimensional solid parts has been devised. The results obtained so far agree with experiment, at least qualitatively.

The number of nodes is still limited to a few thousands, enough to analyse the dynamical behaviour of simple parts of various materials. A very simple user interface allows people with a limited knowledge of continuum mechanics to use it. Students will study how changes the behaviour of a material when the material constants are changed. The main application is to help understand what happens during impact. It is hoped that the software may also be useful for understanding the static behaviour in the same fashion as with traditional finite element methods. The advantage of the method is that it is not limited to elastic behaviour. Viscoelastic and plastic behaviours, with or without an upper yield stress, shear bands and crack formation and propagation are easily simulated.

Crack nucleation and propagation near a notch in a brittle material has been simulated (at least qualitatively) in a realistic manner on a graphical microcomputer with an explicit dynamical finite difference method. Crack propagation was successfully simulated with the Coulomb criterion, without using the concept of stress intensity factor as a fracture criterion. A significant relationship between crack speed and stress intensity factor was not yet found. This may be a question of precision that could be solved by using a finer mesh. Despite the relatively high number of nodes, the stress concentrations are too low and a more powerful microcomputer is necessary for better results. Modifying the fracture criterion to obtain a fracture energy independent of the mesh size will be necessary and the constitutive law may perhaps give a deeper insight on the parameter governing the crack speed. The numerical crack speed, like all theoretically predicted crack velocities<sup>9</sup>, is much higher than found from experimental data and unfortunately the question still remains unsolved.

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