THE USE OF COMPUTER MEANINGS IN TEACHING THE „RULED SURFACES,” CHAPTER OF DESCRIPTIVE GEOMETRY

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Abstract. The “Ruled Surfaces” chapter of Descriptive Geometry, the subject of this work, belongs to the applicable part of this matter and, although it attracts the interest of the student, it is considered as having a high level of difficulty. The presentation in 3D of the ruled surfaces, axonometric, apply the intuition of onlooker and this is why provides a complete sight of the subject however complicate would be. The successive transformation of that, in 2D images, meaning in orthogonal projections, can easily be done by using the AutoCAD 3D or other programs which makes possible to use 3D animation. This manner of presenting simultaneously the two methods to visualize can provide a certain easiness to discern the main orthogonal projections in a unit space and can lead to obtain the capacity to represent in orthogonal projection on any plane of the objects however complicated they are.

The authors also consider that the representation based on the computer of the various types of surfaces, here those called “ruled”, as well as the connection between the examples suggested to be solved and the most famous modern buildings could make this subject more attractive.

Key-words: rolled surfaces, Descriptive Geometry, famous buildings, orthogonal projections, paraboloid, cylindroid, helicoid, hyperboloid.

1. INTRODUCTION

Teaching the Descriptive Geometry is, without any doubt, unless a first for the system of university Romanian education, but a recent attempt to align itself with the last tendencies of education. A particular effort of those who teach is presumed in order to provide such a course of Descriptive Geometry, regarding the adaptation of the means of presenting the matters using an adequate "software", for example the program "AutoCAD 3D". This is why this work is a first step in attempting to build a system of teaching and of evaluation based on the computer.

This article also proposes a review of some buildings that illustrate several types of problems concerning the geometrical representation of “ruled surfaces”. The elements which compose certain surfaces are proposed to be
identified in the assembly of the specified buildings and to be represented in the usual system of projections.

The followings stages were established in the presentation of the subject, in order to fulfill all the requests of a Descriptive Geometry lesson: the view of an existing building, the 3D representation of it, the 3D and 2D view of the constituent elements and finally, the way to form the surface.

2. ABOUT RULLED SURFACES

„Ruled surfaces” are made by moving a straight-line whose ends lay on two curves (or other straight-lines), named directories (as given curves). There are some different surfaces, depending on the geometrical shape of the directories. If the both directories are curves the shape will be a cylindroid, by changing one of the curves in a straight line the surface will become a conoid and finally, if the both directories are straight lines the result will be a hyperbolic paraboloid.

3. EXEMPLES

3.1. Cilindroids

3.1.1. The first example selected to illustrate this subsection of the chapter is a spectacular component of a famous ensemble, the Athens Olympic Sports Complex, designed by one of the greatest builders of our times, architect and engineer at the same time, Santiago Calatrava and built for the 2004 Olympic Games, to celebrate 2800 years since the first Olympic Games. This complex is special because of its technological superiority and its cultural significance. One of the architectural features of the Sports Complex, rather an art work then a building, is the Nations Wall, a monumental sculpture made of 960 tubular steel elements. A battery of motors permits the elements of the Nations Wall to move in a wavelike motion, creating a pleasing effect of light and shadow over the central pedestrian route of the Plaza of Nations. It can also serve as a giant video screen. It is 856 feet long, 65 feet high and raised 3 meters above the ground.

FIGURE 1 AND 2. “Nations Wall” (left) the axonometric view (right)

The following problem it is proposed to be solved according the example above: axonometric representation of the cylindroids is asked, with the first given curve – two semicircles with centers B and D respectively, in a plane
parallel to \([H]\), the second given curve two semicircles with centers \(b\) and \(d\), in the plane \([H]\). The semicircles with centers \(b\) and \(D\) are between plane \([V]\) and a plane parallel to \([V]\) through the points \(A\), \(E\) and \(a\). The semicircles with centers \(B\) and \(d\) are on the other side of the plane through \(A\), \(E\) and \(a\), mentioned above.

13 rulings have to be drawn, from among \(Aa\) will be the first (see fig.2). \(A\) (135, 35,120), \(B\) (110, 35,120), \(C\) (95, 35,120), \(D\) (60, 35,120), \(E\) (35, 35,120).

![FIGURE 3. Cylindroids – the rotation of the surface until the projection to plane \([W]\)](image3)

![FIGURE 4. Cylindroids – the rotation of the surface until the projection to plane \([V]\)](image4)

![FIGURE 5. Cylindroids – the rotation of the surface until the projection to plane \([H]\)](image5)

The problem may be extended by adding the request to repeat the cylindroids surface several times, or by the rotation of the surface with a 90° angle (see Figure 6).

![FIGURE 6. Cylindroids – axonometric representation](image6)

The building which inspired the rotated surface is the wine Complex in Bodegas Ysios - San Sebastian designed by Santiago Calatrava (figure 7)

![FIGURE 7: Wine Complex in Bodegas Ysios - San Sebastian](image7)
3.1.2. The following example was inspired by the quasi similar roofs of the Athens Olympic Stadium and of the Olympic Velodrome. It is composed by a pair of bent “leaves” with a central area of sun-protected laminated glass. Double-inclined arches, 45 meters high, are joined by cables to support the roof. The cables support the transverse ruling that form the surface of the roof. The design shields protects athletes and spectators the same, from the sun and wind.

The request of the problem is to represent the ruled surface that has both curves circle arcs with a radius of 30 and 50mm respectively, in a plane that makes a 60$^\circ$ angle to the plane [H]. Point M (60,40,0) is the middle of the segment which represents the cord sub-stretched by both directories. It is also required the symmetrical surface to this one relative to the plane that includes point N(60,30,0) and is parallel to the plane [V] (see fig.10).

3.1.3. Inaugurated in 2003, the Opera House in Tenerife is an imposing building, whose roof includes a spectacular component, known as „wing“. 17 prefabricated parts, 60 tons in weight and designed to be supported only by 5 points, joint to compose the ensemble. „The wing“ is 197 feet high. The shape of this building is that of two symmetric cylindroids, which makes it ideal for the illustration of this type of ruled surface.
In connection with the figures above, the problem demands to represent the cylindroid that has first given curve an arc on the 45mm radius semicircle and the center S(55,20,0), in a plane parallel to [V], above [H], inscribed between its point of intersection with [H] and point R(25,20,?). The second curve is also a arc, in a plane perpendicular to [H], that makes with the plane of the first given curve an angle of 15°, which belongs to a semicircle with a radius of 70mm, above [H], that intersects the first curve in R. Also it is asked to draw the symmetric surface relative to the plane of the first curve (fig. 11 right).

3.1.4. “Umbraco” (Promenade and Car Park) is part of the complex City of Arts and Sciences in Valencia. The structure is a succession of 55 fixed and 55 mobile steel arches, that cover a botanical garden which is also a promenade placed above the immense area of the car park. Its total length is 320 m, the width is 32.8 and the height (from the lower part of the fixed arch to the top of the mobile one) is 1.8 m.

The same basic idea may be seen in a building of the Athens Olympic Games Complex, 2004: the 100 arcs gallery enclosing an immense semicircular “agora” reflected in a water mirror. It is a spectacular pedestrian way between the two stadiums: the Olympic Stadium and the Velodrome.

Obviously, the successive arcs of the both constructions can be assimilated with the directories of some identically successive cylindroids.
The next problem (fig.15) was inspired by the examples above: the ruled surface supposed to be drawn has an arc which belongs to a semicircle with a 20mm radius, centre S, S (120,120,120) in a plane parallel to [W] and the both tangents connected to this circle arc as the first given curve. The second curve has the same shape as the first, parallel to it, at the 20 mm distance, 20mm downer then the first arc. It could also demand the representation of the symmetric surface to this one relative to the plane of the second directrix connected to the end of this and even more, other ruled surfaces, symmetric to those described before, relative to the plane of the last figured curve.

3.2. Hyperboloids (Particular Cylindroids)

3.2.1. The hyperboloid encountered in every-day life in urban areas, is usually connected with industrial constructions and it often has a negative visual impact. However, some architects have changed this traditional opinion about these geometrical bodies and surprised us, creating exceptional shapes.

The Brasilia “Cathedral” is the creation of the famous architect Oscar Niemayer. The building, inaugurated in 1970, lay on 16 identical concrete columns which symbolize the human hands raised to the sky (figure 16).
The problem demands the representation of a hyperboloid with both basic curves circles. The first curve is the circle with center S and radius 50 mm in [H] and the second is a circle with radius 30 and centre T in a plane parallel to [H] 90 mm distanced from the first. The divergence between the horizontal projections of the ends of the directories is 120° for each curve.

3.3. Conoids

3.3.1. The first conoid which it demand to be represented is based on the building of the Milwaukee Art Museum, built in 1957 and renovated in 1975 and 2001, when new architectural elements designed by Calatava were added to it. The signature element of the Museum is The Brise Soleil, a giant movable sunshade, consisting of 72 fins of variable length (between 6,60 and 31,00 meters) forming two wings that could be manually controlled to lift up and over the pavilion, controlling the temperature and sunlight inside. The fins are made of rectangular metallic profiles with rounded corners, with one fixed dimension of 330 mm and a variable one of between 400 and 1000 mm. The “Civil Engineering Award“ by the American Society of Civil Engineering was conferred to Santiago Calatrava in 2002 for the MAM extension.

![Calatrava’s “Brise Soleil”](image)

3.3.2. The next example was inspired by one of the most famous buildings in the world: the Opera House in Sydney, Australia. Created by the Danish architect Jorn Utzon, it was start to be erected in March 1959 and was finally completed in 1973. The complex was modified several times, enabling it to contain five theatres. The impressive building has 1000 rooms, covers 1,8 hectares, is 185 m in length, 120m wide and rises to a height of 67 m above sea level. The spherical surfaces of the roof were approximated into plane, between the arches considered as basic curves of the surfaces.

![Axonometric view](image)
The initial data of one of the proposed problems, inspired by the above-mentioned construction are as request to represent 3 conoids. All the surfaces have as the first directories 2 symmetric arcs for each conoid, in a plane inclined at $60^\circ$ to [H]. The arcs of the first directories are 65 mm radius for the first conoid, 80 mm radius for the second and 95 mm radius for the third. The segment between the ends of the first directories is ST for the first conoid, $S_1T_1$ for the second and $S_2T_2$ for the third. $S(160,55,0)$ $T(160,105,0)$ $S_1(110,40,0)$ $T_1(110,120,0)$ $S_2(60,25,0)$ $T_2(60,135,0)$. The second directories are the segment $S_1T_1$ for the first conoid, $S_2T_2$ for the second and $S_3T_3$ for the third. $S_3(10,10,0)$ and $T_3(10,150,0)$. (figure 21)

3.3.3. The right side of the picture 20 above has inspired a different problem whose request is to represent two conoids as follows:

The first curve of the first conoid belongs to a plane $60^\circ$ inclined to the horizontal, composed of two arcs within a 65 mm radius circle, through points S and T, above [H] with $S(90,15,0)$ and $T(90,65,0)$ and the second given curve is the segment MN, which lay on a line perpendicular to [V]. $M(50,25,50)$, $N(50,55,50)$. The second conoid is symmetric the first, relative to a plane parallel to [W] through M and N. (figure 22)
3.3.4. The third problem based on the example of the roof of the Olympic stadium of Athens (fig.9) demands to draw the conoid with the 30mm semicircle and M (90, 50, 0) as the center which belong to a plane inclined at 60° relative to [H] as the first diretrix and the segment ST, 20mm in length which belongs to a straight line perpendicular to [W] as the second. S(75,30,-20). It is also asked for the symmetric surface in relation with the plane through the points S and T, parallel to [V] and for the intersections of the conoids with the plane[H].

3.4. Helicoids (Particular Conoids)

3.4.1. “Turning torso” is a “sky-scrapers” whose 190 m high make it the highest building in Scandinavia and the second highest among apartment-blocks in Europe. It was inaugurated in August 2005, in Malmo, Sweden. The upper part of the building is rotated at 90° relative to the lower. In fact, it is a helicoid with an uncommonly large difference between the rulings which belong to the successive floors of the building.
Concerning the example it is demand to represent the axonometric representation of the group of 4 identical helicoids (see fig. 25), whose directories belong to a cylindrical surface which could be drawn as following:

- surface I, the first given curve starts from point A(50,50,0) and the second from B(20,50,0).
- surface II, the first starts from B and the second from C (20, 20, 0)
- surface III, the first starts from C and the second from D (50, 20, 0)
- surface IV, the first curve starts from point D and the second from A(50,50,0).

The first 4 rulings of each vertical surface belong to plane [H] and make a square ABCD which 30 mm side. The distance between two successive planes is 30 mm and each plane is rotated at 30° in relation with the previous.

3.4.2. The future highest Chicago skyscraper will be a 115 stories building, which would reach 1,458 feet then add a 2,000 feet steel spire. The shape of the building will be a helicoid whose diameter decrease up to the minimum at the top of the sharp steel spire.
FIGURE 27. “Fordham spire”, Chicago, the model (left), the axonometric view (right)

FIGURE 28. Axonometric view (left), vertical orthogonal projection (middle left), rotation of the axonometric view (middle right), horizontal orthogonal projection (right)

The request of following problem which the “Chicago spire” was illustrated by is to represent the helicoid which has a straight line perpendicular to [H] through the point \( H (60,60,0) \) as the first given curve and a curve which belongs to a con with 48mm radius through \( H (60, 60, 0) \) as the second. The distance between two successive ribs is 5mm and its length is 2mm less than the previous. Each ruling is rotate with a \( 45^0 \) angle than the previous.

3.5. Hyperbolic Paraboloids

3.5.1. The first example belongs to a group of several constructions whose shapes are hyperbolic paraboloid. There are the three bridges over the Hoofdvaart canal in Haarlemmermeer, Holland. The bridge elements are supported by cables attached to a main pylon differently positioned for each of them. The cables, easily discernible in the surrounding landscape, illustrate very well the ruling of the ruled surface further analyzed. One of the bridges inspired the first problem which is proposed to be solved.

FIGURE 30. The real model (left), the axonometric view of the parabolic surface (middle), a vertical projection on the [W] projection plane (right).
The request of the problem is to build a hyperbolic paraboloid which the first given curve is the straight line AB, the second is CD, with A, B and D in [H] and C in [V]. AB is parallel to Oy, AB=45mm, BD parallel to Ox, BD=30mm. The coordinates of the point A are A (50, 10, ?) and of C (35, ?, 79)(figure 30).

Some steps are proposed to be followed to solve problem:
1. to place the points A, B, C and D within the axis
2. to joint these points in order to trace the outline of the surface
3. to divide the both directories in an identical number of equal segments
4. to trace the straight lines; taking account the visibility of the shape, the first ruling will be BC.

3.5.2. The application might also require the representation of the surface symmetrical to the base plane (see figure 31). The solution to this last requirement was inspired by the shade of the bridge reflected on the surface of the canal water.

4. CONCLUSIONS

The new manner of teaching provides not only a maximum efficiency of the study, but essential more, it radically changes the mentality and the attitude of the students, to be involved and responsible in the relationship with their own training to fulfill the university conditions and stimulates simultaneously the students’ creativity.

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